AN APPROXIMATE CALCULATION OF THE EXTENT TO WHICH SHOCK WAVES ARE WEAKENED BY PERMEABLE BARRIERS

B. E. Gel'fand and S. M. Frolov UDC 532.593

In order to extinguish shock waves (SW) in gasdynamic laser pulse systems we make use of permeable shields in the form of wire grids or perforated barriers [1]. A review of the methods to calculate the weakening effect of such shields with respect to SW is presented in [2]. In the present study we offer a semiempirical method of calculating SW attenuation by means of permeable barriers, said method based on an idea raised in [3].

1. Weakening of a SW with a Single Permeable Barrier. Let us examine the evolution of a plane SW with a stepped profile subsequent to entering a nonmoving permeable shield positioned in the (x_1, x_2) segment. Without going into the details of the wave-field diagram as the SW interacts with the barrier, let us hypothecate that the following condition, valid for an overtaking characteristic, is satisfied at the SW passing through the shield:

$$(u + a)\, dp/dx + a\rho\, (u + a)\, du/dx = [(\gamma - 1)\, u - a]\, f(\rho u^2/2), \tag{1.1}$$

where u is the velocity of gas; a is the speed of sound; p is the pressure; ρ is the density; γ is the heat-capacity ratio; $x_1 \le x \le x_2$ denotes the distance covered by the SW; f is a parameter (constant at large Reynolds numbers) characterizing the aerodynamic resistance of a unit of shield length. Once we have substituted into (1.1) the relationships at the compression discontinuity, expressed in terms of the Mach number M of the passing wave, we obtain

$$\varphi(M, \gamma)\, dM/dx = -f/2 \tag{1.2}$$

for $M(x_1) = M_0$ (M_0 is the Mach number of the original SW). The solution for Eq. (1.2) for the intensity of the SW at the outlet from the shield, i.e., $M(x_2) = M_1$, has the form

$$G\,(M_0, \gamma) - G\,(M_1, \gamma) = f\,(x_2 - x_1)/2$$
$$\left(G\,(M_0, \gamma) - G\,(M_1, \gamma) = \int\limits_{M_1}^{M_0} \varphi\,(M, \gamma)\, dM\right). \tag{1.3}$$

It is quite cumbersome to calculate the integrals, and we attempted to attain it with the numerical Simpson method. For convenience in the calculations we derived an approximation of the function G(M) for $\gamma = 1.4$

$$G\,(M) = 4\,\frac{0{,}4M - 1}{M^2 - 1} + 4\ln\,(M^2 - 1) + 0.8\ln\frac{M + 1}{M - 1}. \tag{1.4}$$

The deviation of function (1.4) from the true values does not exceed 5% when M = 1.01-4.

On passing to the limit $x_2 \to x_1$ let the quantity $f(x_2 - x_1)/2$ in (1.3) strive toward a constant value of η, characterizing the attenuation of a SW by an isolated permeable barrier. For a three-dimensional wire grid with a specific permeability Ω (the ratio of the free cross-sectional area of one layer of the grid to the overall frontal area of the transverse cross section) and the grid interval s in the longitudinal direction $f = C_D(1 - \Omega)/s$ (C_D is the coefficient of aerodynamic drag for the structural element of the grid layer). When $x_2 - x_1 = (n - 1)s \to 0$ (n represents the number of layers), $\eta \to C_D(1 - \Omega)$. As we can see, η is determined from the geometric characteristics of the barrier: the permeability Ω and the shape of the structural element, namely: C_D. Since the C_D factor of poorly steamlined bodies in a free flow at high Reynolds numbers are close in magnitude to one another [4], and since the thickness of the barriers has only a slight effect on the efficiency of SW extinction [5], it should be expected that all of the barriers will basically be characterized only by Ω.

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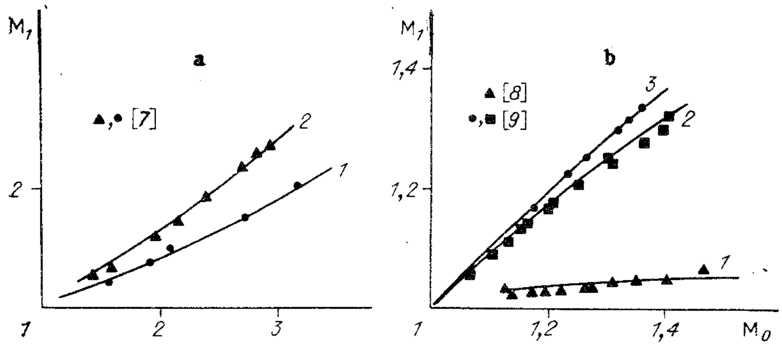

Fig. 1

In order to verify the validity of the simplifying assumptions, we will use the experiments from [1, 5-11] into the extinction of SW by perforated barriers or grids. As an example, in Fig. 1 we have plotted the experimental data from [7-9] for barriers of various permeabilities. The experimental points have been processed in accordance with (1.3) and (1.4) by the method of least squares, which made it possible to determine η for a barrier with a given permeability. The parameter η is constant along each curve [Fig. 1a: 1) $\Omega = 0.113$, $\eta = 4.5$ [7]; 2) 0.347, 2.2 [7]; Fig. 1b: 1) $\Omega = 0.0385$, $\eta = 40$ [8]; 2) 0.55, 1.6 [9]; 3) 0.7, 0.58 [9]]. Thus, the hypothesis as to the independence of η from M_0 is confirmed. Analogous calculations have been carried out on the basis of the data from [1, 5, 6, 10, 11]. In Fig. 2 we see the final relationship of η to Ω. All of the points are grouped near the curve described by the equation

$$\eta = 26(\Omega^{-0,1} - 1). \tag{1.5}$$

Unlike [1, 6-9, 11] in [5, 10] the attenuation of a triangular SW is subjected to examination, and in [10] the SW has been generated by means of explosive charges. The wave profile apparently is inconsequential from the standpoint of the final results because of the relatively small thickness of the barriers being used here, in comparison to the wavelength. The measurements in [1, 5, 7, 11] were carried out for $p_0 = 0.1$ MPa, while in [6] they were carried out for $p_0 = 5 \cdot 10^{-4}$ MPa (p_0 is the initial pressure). The fact that the data from various authors are grouped about a common curve independent of the configuration of the perforated barriers and grids speaks in favor of the validity of the idea from [3] according to which the shock front moving through the blocked space experiences virtually no wake effects.

2. Attenuation of a SW in a Cascade of Permeable Barriers. The efficiency of SW attenuation is altered when several permeable barriers are installed in the channel. With small distances between barriers having a permeability of Ω_i the efficiency of attenuation SW at the second, third, i-th, etc., barriers is smaller than $\eta_2(\Omega_2)$, $\eta_3(\Omega_3)$, $\eta_i(\Omega_i)$, etc. [9]. With larger distances the plane shock front formed after passage of the previous barrier can be regarded as the initial perturbation for the subsequent disturbance. In this event, the intensity of the shock wave passing through n permeable barriers is found from the conditions

$$\begin{aligned} G(M_1) &= G(M_0) - \eta_1, \\ G(M_2) &= G(M_1) - \eta_2, \\ &\dots\dots\dots\dots \\ G(M_n) &= G(M_{n-1}) - \eta_n, \end{aligned} \tag{2.1}$$

where M_i and η_i ($i = 1, 2, \ldots, n$) are, respectively, the Mach number of the SW passing through the i-th barrier, and its attenuation parameter. It is easy to see from (2.1) that $G(M_n) = G(M_0) - \sum_{i=1}^{n} \eta_i$, i.e., the total attenuation parameter $\eta_\Sigma = \sum_{i=1}^{n} \eta_i$. For identical barriers

$$\eta_\Sigma = n\eta. \tag{2.2}$$

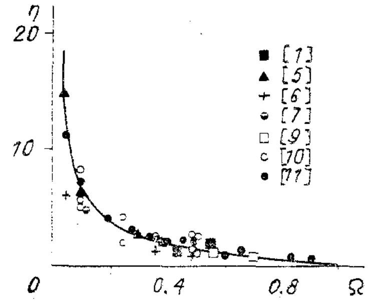

Fig. 2

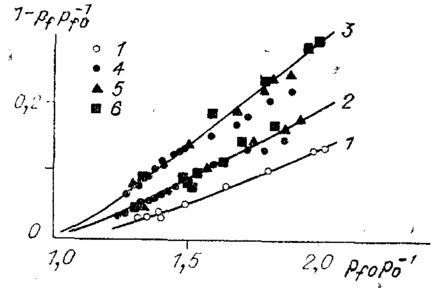

Fig. 3

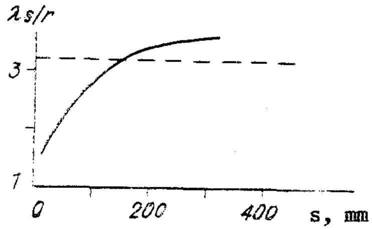

Fig. 4

Figure 3 shows a comparison of the theoretical relationship (2.2) with the experimental data from [9] on the attenuation of an air SW with a single circular insert (1), a cascade of two inserts (2), and four (3) circular inserts with $\Omega = 0.55$ ($\eta \approx 1.6$). The distances between these inserts is s = 100 (4), 200 (5), and 400 mm (6). The pressure ratio p_{f0}/p_0 at the incident SW is plotted along the axis of abscissas, while along the ordinate axis we see a plot of the parameter characterizing the attenuation of the SW (p_f if the pressure in the SW passing through the shield). We can see from Fig. 3 that (2.2) offers a satisfactory prognosis. In this case, as the interval s between the barrier increases, the accuracy of relationship (2.2) rises. As for s = 100 mm the experimental points fall below the theoretical curve, which suggests the presence of a mutual barrier effect.

The following relationship, having no theoretical foundation, is proposed in [12] for the attenuating effect of n permeable barrier on the SW:

$$\beta_\Sigma = \beta^n \tag{2.3}$$

(β is the attenuation factor for the excess pressure in the SW as it passes through a single barrier and β_Σ is the total factor). It is indicated in [12] that (2.3) is valid only in the case of virtually isolated barriers, i.e., for those at considerable distances from one another. Let us find a relationship between η and β. When $M - 1 \ll 1$, Eq. (1.2) has the following solution [13]:

$$\eta = \frac{\gamma+1}{2}\left(\frac{1}{M_1 - 1} - \frac{1}{M_0 - 1}\right).$$

Since $\beta = \Delta p_f/\Delta p_{f0}$ (Δp is the excess pressure at the front of the SW), then

$$\beta = \left[1 + \frac{2}{\gamma+1}(M_0 - 1)\,\eta\right]^{-1}. \tag{2.4}$$

For n identical barriers, according to (2.3) we have $\beta_\Sigma = \beta^n \approx \left[1 + \frac{2}{\gamma+1}(M_0 - 1)\,n\eta\right]^{-1}$, i.e.,

$$\beta_\Sigma = \beta(n\eta), \tag{2.5}$$

and conditions (2.2) and (2.3) when $M_0 \approx 1$ are identical. Unlike η, the coefficient β according to (2.4) depends on the intensity of the incident wave. This makes its utilization inconvenient in practical calculations. When linear approximation is not permissible, the relationship among β, M_0, and η is more complex in nature than in (2.4). For purposes of illustrating the relationship between β and M_0, Table 1 gives the values of β for a barrier

TABLE 1

M_0	β
1,1	0,896
1,2	0,838
1,3	0,797
1,4	0,773
1,5	0,742
2	0,680
3	0,656

TABLE 2

M_0	$\beta(2\eta)$	β^2	Δ
1,1	0,818	0,803	0,018
1,2	0,717	0,702	0,021
1,3	0,659	0,635	0,036
1,4	0,603	0,598	0,008
1,5	0,569	0,551	0,032
2	0,478	0,463	0,030
3	0,439	0,430	0,020

with $\Omega = 0.55$ ($\eta \approx 1.6$). We can see from Table 1 that when $M_0 > 1$, β depends significantly on M_0. The data in Table 2 show the extent to which relationships (2.3) are valid for a cascade consisting of two "independent" barriers with identical $\Omega = 0.55$. The magnitude of Δ characterizes the relative deviation of (2.3) from (2.5), a consequence of (2.2). We can see from Table 2 that over a broad range of M_0 numbers, Eq. (2.3) yields a rather precise prognosis.

One of the most important questions in this problem deals with the smallest distance between the barriers, at which these barriers may be held to be "independent." In order to solve this problem we will make use of the SW attenuation data in elongated permeable shields. It would be natural to expect that a cascade of circular barriers, each of which is determined by the attenuation parameter $\eta(\Omega)$, will function as a rough tube segment characterized by some value for the coefficient of hydraulic drag λ. As demonstrated in [14], the right-hand side of (1.3) in the case of a rough tube with annular inserts $f(x_2 - x_1)/2 \simeq \lambda(x_2 - x_1)/r$ (r is the radius of the tube). Then, for the two adjacent "independent" barriers in such a shield the following condition should be satisfied; namely:

$$\lambda(s)s/r = 2\eta, \tag{2.6}$$

where s is the unknown distance between the barriers. We will test the validity of (2.6) by using the data from [9] and the results obtained in the measurement of the hydraulic resistance of tubes with circular inserts, as given in [15]. Figure 4 shows the dependence of λsr^{-1} on the distance s between identical barriers with $\Omega = 0.55$ ($\eta = 1.6$), as used in the experiments conducted in [9]. We see that with an increase in s the parameter λsr^{-1} increases, reaching values of 2η (the dashed line) at a distance of $s \approx 150$ mm. This result is in satisfactory agreement with the experimental data from [9] (see Fig. 3). The fact that λsr^{-1} when $s > 150$ mm is somewhat higher than 2η can be explained by the increased contribution resulting from the resistance of the smooth tube segment between the barriers. In this case the increase in s will result in greater attenuation of the SW than when we take into consideration only the drag resulting from the shape of the barrier.

Thus, relationships (1.3)-(1.5) allow us to calculate the attenuation of a SW by a single permeable barrier, while (1.3)-(1.5), (2.2) enable us to calculation this attenuation for a casade of barriers.

LITERATURE CITED

1. K.-O. Tong, C. G. Knight, and B. N. Srivastava, "Interaction of weak shock waves with screens and honeycombs," AIAA J., 18, No. 11 (1980).
2. E. F. Zhigalko, The Dynamics of Shock Waves [in Russian], Izd. LGU, Leningrad (1987).
3. G. B. Whithem, Linear and Nonlinear Waves, Wiley, New York (1974).
4. A. I. Bogomolov, V. S. Borovkov, and F. G. Mairanovskii, High-Speed Flows with a Free Surface [in Russian], Stroiizdat, Moscow (1979).
5. B. E. Gelfand, S. P. Medvedev, A. N. Polenov, et al., "Interaction of nonstationary pressure waves with perforated partitions," Arch. Combustionis, 7, No. 1-2 (1987).
6. A. V. Bondarenko and M. A. Ibragim, "Measuring the velocity of a shock wave established on interaction with barriers in the form of diaphragm channels," Izv. Akad. Nauk SSSR, MZhG, No. 3 (1967).
7. Yu. P. Kostochko, "Interaction of shock waves with a permeable surface," in: Transactions of the Tomsk State University, Scientific Research Institute, PMM, Vol. 5 (1974).
8. C. G. Dain and J. P. Hodson, "Generation of weak shock waves in a shock tube," Aeronaut. Quart., 25, No. 2 (1974).

9. Y. Mori, K. Hijikata, and T. Shimizu, "Attenuation of shock wave by multiorifice," in: Proc. 10th Symp. (Int.) on Shock Tubes and Waves, Kyoto (1975).
10. V. N. Mineev, V. E. Klapovskii, B. V. Matseevich, et al., "Attenuation of the effect of an air shock wave by means of perforated barriers," in: 5th Int. Symp. on the Explosive Working of Metals, Gottwaldov (1982).
11. V. D. Shevtsov, "Interaction of shock waves with permeable barriers," Dep. Paper No. 1192-82, VINITI, March 16, 1982.
12. W. E. Baker, P. A. Cox, P. S. Westine, et al., Explosion Hazards and Evaluation, Elsevier, Amsterdam (1983).
13. N. Zloch, "Shock attenuation in beds of granular solids," Arch. Mech. Stosow., 28, No. 5-6 (1976).
14. B. E. Gel'fand, S. M. Frolov, S. P. Medvedev, et al., "Extinction of shock waves in channels. Rough tubes," Preprint No. 120, Akad. Nauk SSSR, OIKhF, Chernogolovka (1989).
15. R. Koch, "Druckverlust und Wärmeübergang bei verwirbelter Strömung," VDI-Forschungsheft., No. 469 (1958).

THEORY OF THE HARDENING OF BINARY MELTS WITH AN EQUILIBRIUM TWO-PHASE ZONE

Yu. A. Buevich, L. Yu. Iskakova, and V. V. Mansurov UDC 536.421.4

Descriptions of processes involved in directed crystallization is normally accomplished on the basis of concepts dealing with the existence of a clearly defined phase-transition front and reduces to the solution of various versions of the Stefan problem [1-3]. If the liquid that is subjected to hardening is one consisting of numerous components, the motion of the front is accompanied by a redistribution of the composition of the phases, and in addition to the equations of heat conduction, it is necessary also to deal with the equations of diffusion and the relationship between the temperature of the phase transition and the composition of the melt, or its dependence on the composition of the solution near the front. Under specific conditions the effective temperature of the liquidus ahead of the front proves to be higher than the temperature of the liquid phase, i.e., a metastable supercooled zone is formed [4]. The same situation is encountered in the hardening of supercooled single- or multicomponent liquids.

In the metastable region conditions prevail for the growth of the solid phase nuclei, generated spontaneously or in the impurity crystallization cores. Moreover, the front becomes morphologically unstable, which may lead to the development of a system of dendrites. Either mechanism enhances the appearance of a transition two-phase zone (in which the liquid and solid phases coexist) ahead of the front, as well as to the partial removal of the supercooling. In the general case this zone is thermodynamically in a state of nonequilibrium, and its characteristics determine the relationship between the kinetic processes of formation and the growth of the solid-phase elements, as well as the velocity at which the front is displaced. Supercooling ranging from the very lowest to several tens of degrees has been experimentally established [5-8].

The traditional frontal formulation describes approximately the situation in which the two-phase zone is almost entirely absent, which is characteristic of pure liquids under conditions in which the morphological instability stimulates development of cellular structures, but no dendrites (the majority of semiconductor and certain metal melts). In the opposite extreme case (melts with nuclei or crystallization catalysts, liquid seals, true aqueous solutions) it is permissible to use an approximation of an equilibrium two-phase zone in which supercooling has been entirely removed [9-12]. Then, in connection with the fact that the literature is full of an excess of categorically extreme assertions either as to the significance of concentration supercooling in those cases in which it should appear

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